

Scientific Form

The last form is called the **scientific form** of the number. There is **one** nonzero digit to the left of the decimal point and the absolute value of the exponent on 10 records the number of places the original decimal point was moved to the **left**.

$$\begin{aligned} 0.00059 &= \frac{0.0059}{10} = \frac{0.0059}{10^1} = 0.0059 \times 10^{-1} \\ &= \frac{0.059}{100} = \frac{0.059}{10^2} = 0.059 \times 10^{-2} \\ &= \frac{0.59}{1000} = \frac{0.59}{10^3} = 0.59 \times 10^{-3} \\ &= \frac{5.9}{10,000} = \frac{5.9}{10^4} = 5.9 \times 10^{-4} \end{aligned}$$

There is **one** nonzero digit to the left of the decimal point and the absolute value of the exponent of 10 records the number of places the original decimal point was moved to the **right**.

Scientific Notation

Numbers written in scientific form are also said to be written using scientific notation. In **scientific notation**, a number is written as the product of a number between and including 1 and 10 (1 is included, 10 is not) and some power of 10.

Writing a Number in Scientific Notation

To write a number in scientific notation:

1. Move the decimal point so that there is one nonzero digit to its left.
2. Multiply the result by a power of 10 using an exponent whose absolute value is the number of places the decimal point was moved. Make the exponent positive if the decimal point was moved to the left and negative if the decimal point was moved to the right.

3 Sample Set A

Write the numbers in scientific notation.

Example 1

981

The number 981 is actually 981., and it is followed by a decimal point. In integers, the decimal point at the end is usually omitted.

$$981 = 981. = 9.81 \times 10^2$$

The decimal point is now two places to the left of its original position, and the power of 10 is 2.

Example 2

$$54.066 = 5.4066 \times 10^1 = 5.4066 \times 10$$

The decimal point is one place to the left of its original position, and the power of 10 is 1.

Example 3

$$0.000000000004632 = 4.632 \times 10^{-12}$$

The decimal point is twelve places to the right of its original position, and the power of 10 is -12 .

Example 4

$$0.027 = 2.7 \times 10^{-2}$$

The decimal point is two places to the right of its original position, and the power of 10 is -2 .

4 Practice Set A

Write the following numbers in scientific notation.

| | |
|--|-----------------------------|
| Exercise 1 346 | <i>(Solution on p. 10.)</i> |
| Exercise 2 72.33 | <i>(Solution on p. 10.)</i> |
| Exercise 3 5387.7965 | <i>(Solution on p. 10.)</i> |
| Exercise 4 87,000,000 | <i>(Solution on p. 10.)</i> |
| Exercise 5 179,000,000,000,000,000,000 | <i>(Solution on p. 10.)</i> |
| Exercise 6 100,000 | <i>(Solution on p. 10.)</i> |
| Exercise 7 1,000,000 | <i>(Solution on p. 10.)</i> |
| Exercise 8 0.0086 | <i>(Solution on p. 10.)</i> |
| Exercise 9 0.000098001 | <i>(Solution on p. 10.)</i> |
| Exercise 10 0.0000000000000000054 | <i>(Solution on p. 10.)</i> |
| Exercise 11 0.0000001 | <i>(Solution on p. 10.)</i> |
| Exercise 12 0.00000001 | <i>(Solution on p. 10.)</i> |

5 Scientific Form to Standard Form

A number written in scientific notation can be converted to standard form by reversing the process shown in Sample Set A.

Converting from Scientific Notation

To convert a number written in scientific notation to a number in standard form, move the decimal point the number of places prescribed by the exponent on the 10.

Positive Exponent Negative Exponent

Move the decimal point to the right when you have a positive exponent, and move the decimal point to the left when you have a negative exponent.

6 Sample Set B

Example 5

$$4.673 \times 10^4.$$

The exponent of 10 is 4 so we must move the decimal point to the right 4 places (adding 0's if necessary).

$$\underline{4.6730} \times 10^4 = 46730$$

Example 6

$$2.9 \times 10^7.$$

The exponent of 10 is 7 so we must move the decimal point to the right 7 places (adding 0's if necessary).

$$2.9 \times 10^7 = 29000000$$

Example 7

$$1 \times 10^{27}.$$

The exponent of 10 is 27 so we must move the decimal point to the right 27 places (adding 0's without a doubt).

$$1 \times 10^{27} = 1,000,000,000,000,000,000,000,000,000$$

Example 8

$$4.21 \times 10^{-5}.$$

The exponent of 10 is -5 so we must move the decimal point to the left 5 places (adding 0's if necessary).

$$4.21 \times 10^{-5} = 0.0000421$$

Example 9

$$1.006 \times 10^{-18}.$$

The exponent of 10 is -18 so we must move the decimal point to the left 18 places (adding 0's if necessary).

$$1.006 \times 10^{-18} = 0.0000000000000000001006$$

7 Practice Set B

Convert the following numbers to standard form.

Exercise 13

$$9.25 \times 10^2$$

(Solution on p. 10.)

Exercise 14

$$4.01 \times 10^5$$

(Solution on p. 10.)

Exercise 15

$$1.2 \times 10^{-1}$$

(Solution on p. 10.)

Exercise 16

$$8.88 \times 10^{-5}$$

(Solution on p. 10.)

8 Working with Numbers in Scientific Notation

Multiplying Numbers Using Scientific Notation

There are many occasions (particularly in the sciences) when it is necessary to find the product of two numbers written in scientific notation. This is accomplished by using two of the basic rules of algebra.

Suppose we wish to find $(a \times 10^n)(b \times 10^m)$. Since the only operation is multiplication, we can use the commutative property of multiplication to rearrange the numbers.

$$(a \times 10^n)(b \times 10^m) = (a \times b)(10^n \times 10^m)$$

Then, by the rules of exponents, $10^n \times 10^m = 10^{n+m}$. Thus,

$$(a \times 10^n)(b \times 10^m) = (a \times b) \times 10^{n+m}$$

The product of $(a \times b)$ may not be between 1 and 10, so $(a \times b) \times 10^{n+m}$ may not be in scientific form. The decimal point in $(a \times b)$ may have to be moved. An example of this situation is in Sample Set C, problem 2.

9 Sample Set C

Example 10

$$\begin{aligned} (2 \times 10^3)(4 \times 10^8) &= (2 \times 4)(10^3 \times 10^8) \\ &= 8 \times 10^{3+8} \\ &= 8 \times 10^{11} \end{aligned}$$

Example 11

$$\begin{aligned} (5 \times 10^{17})(8.1 \times 10^{-22}) &= (5 \times 8.1)(10^{17} \times 10^{-22}) \\ &= 40.5 \times 10^{17-22} \\ &= 40.5 \times 10^{-5} \end{aligned}$$

We need to move the decimal point one place to the **left** to put this number in scientific notation. Thus, we must also change the exponent of 10.

$$40.5 \times 10^{-5}$$

$$4.05 \times 10^1 \times 10^{-5}$$

$$4.05 \times (10^1 \times 10^{-5})$$

$$4.05 \times (10^{1-5})$$

$$4.05 \times 10^{-4}$$

Thus,

$$(5 \times 10^{17})(8.1 \times 10^{-22}) = 4.05 \times 10^{-4}$$

10 Practice Set C

Perform each multiplication.

Exercise 17

$$(3 \times 10^5)(2 \times 10^{12})$$

(Solution on p. 10.)

Exercise 18

$$(1 \times 10^{-4})(6 \times 10^{24})$$

(Solution on p. 10.)

Exercise 19

$$(5 \times 10^{18})(3 \times 10^6)$$

(Solution on p. 10.)

Exercise 20

$$(2.1 \times 10^{-9})(3 \times 10^{-11})$$

(Solution on p. 10.)

Exercise 36

The mass of an amoeba is about 0.000004 gram.

Exercise 37

Cells in the human liver have masses of about 0.000000008 gram.

(Solution on p. 11.)

Exercise 38

The human sperm cell has a mass of about 0.000000000017 gram.

Exercise 39

The principal protein of muscle is myosin. Myosin has a mass of 0.000000000000000103 gram.

(Solution on p. 11.)

Exercise 40

Amino acids are molecules that combine to make up protein molecules. The amino acid tryptophan has a mass of 0.00000000000000000000340 gram.

Exercise 41

An atom of the chemical element bromine has 35 electrons. The mass of a bromine atom is 0.0000000000000000000000031 gram.

(Solution on p. 11.)

Exercise 42

Physicists are performing experiments that they hope will determine the mass of a small particle called a neutrino. It is suspected that neutrinos have masses of about 0.000000000000000000000000000001 gram.

Exercise 43

The approximate time it takes for a human being to die of asphyxiation is 316 seconds.

(Solution on p. 11.)

Exercise 44

On the average, the male housefly lives 1,468,800 seconds (17 days).

Exercise 45

Aluminum-26 has a half-life of 740,000 years.

(Solution on p. 11.)

Exercise 46

Manganese-53 has a half-life of 59,918,000,000,000 seconds (1,900,000 years).

Exercise 47

In its orbit around the sun, the earth moves a distance one and one half feet in about 0.0000316 second.

(Solution on p. 11.)

Exercise 48

A pi-meson is a subatomic particle that has a half-life of about 0.000000261 second.

Exercise 49

A subatomic particle called a neutral pion has a half-life of about 0.0000000000000001 second.

(Solution on p. 11.)

Exercise 50

Near the surface of the earth, the speed of sound is 1195 feet per second.

For the following problems, convert the numbers from scientific notation to standard decimal form.

Exercise 51

The sun is about 1×10^8 meters from earth.

(Solution on p. 11.)

Exercise 52

The mass of the earth is about 5.98×10^{27} grams.

Exercise 53

Light travels about 5.866×10^{12} miles in one year.

(Solution on p. 11.)

Exercise 54

One year is about 3×10^7 seconds.

Exercise 55

Rubik's cube has about 4.3×10^{19} different configurations.

(Solution on p. 11.)

Exercise 56

A photon is a particle of light. A 100-watt light bulb emits 1×10^{20} photons every second.

Exercise 57*(Solution on p. 11.)*

There are about 6×10^7 cells in the retina of the human eye.

Exercise 58

A car traveling at an average speed will travel a distance about equal to the length of the smallest fingernail in 3.16×10^{-4} seconds.

Exercise 59*(Solution on p. 11.)*

A ribosome of **E. coli** has a mass of about 4.7×10^{-19} grams.

Exercise 60

A mitochondrion is the energy-producing element of a cell. A mitochondrion is about 1.5×10^{-6} meters in diameter.

Exercise 61*(Solution on p. 11.)*

There is a species of frogs in Cuba that attain a length of at most 1.25×10^{-2} meters.

Perform the following operations.

Exercise 62

$$(2 \times 10^4) (3 \times 10^5)$$

Exercise 63*(Solution on p. 11.)*

$$(4 \times 10^2) (8 \times 10^6)$$

Exercise 64

$$(6 \times 10^{14}) (6 \times 10^{-10})$$

Exercise 65*(Solution on p. 11.)*

$$(3 \times 10^{-5}) (8 \times 10^7)$$

Exercise 66

$$(2 \times 10^{-1}) (3 \times 10^{-5})$$

Exercise 67*(Solution on p. 11.)*

$$(9 \times 10^{-5}) (1 \times 10^{-11})$$

Exercise 68

$$(3.1 \times 10^4) (3.1 \times 10^{-6})$$

Exercise 69*(Solution on p. 11.)*

$$(4.2 \times 10^{-12}) (3.6 \times 10^{-20})$$

Exercise 70

$$(1.1 \times 10^6)^2$$

Exercise 71*(Solution on p. 11.)*

$$(5.9 \times 10^{14})^2$$

Exercise 72

$$(1.02 \times 10^{-17})^2$$

Exercise 73*(Solution on p. 11.)*

$$(8.8 \times 10^{-50})^2$$

Exercise 74

If Mount Kilimanjaro was 1,000,000 times as high as it really is, how high would it be? (See problem 1.)

Exercise 75*(Solution on p. 11.)*

If the planet Mars was 300,000 times as far from the sun as it really is, how far from the sun would it be? (See problem 2.)

Exercise 76

If 800,000,000 of the smallest insects known were lined up head to tail, how far would they stretch? (See problem 5.)

Exercise 77*(Solution on p. 11.)*

If Rhea, the moon of Saturn, had a surface area 0.00000000002 of its real surface area, what would that surface area be? (See problem 8.)

Exercise 78

If the star Epsilon Aurigae B had a surface area 0.005 of its real surface area, what would that surface area be? (See problem 9.)

Exercise 79*(Solution on p. 11.)*

If the mass of all the galaxies in the constellation Virgo was only 0.000000000000000000000003 of its real mass, what would that mass be? (See problem 15.)

Exercise 80

What is the mass of 15,000,000,000,000 bromine atoms? (See problem 21.)

12 Exercises for Review**Exercise 81***(Solution on p. 12.)*

(here¹) What integers can replace x so that the statement $-6 < x < -2$ is true?

Exercise 82

(here²) Simplify $(5x^2y^4)(2xy^5)$

Exercise 83*(Solution on p. 12.)*

(here³) Determine the value of $-[-(-|-5|)]$.

Exercise 84

(here⁴) Write $\frac{x^3y^{-5}}{z^{-4}}$ so that only positive exponents appear.

Exercise 85*(Solution on p. 12.)*

(here⁵) Write $(2z + 1)^3(2z + 1)^{-5}$ so that only positive exponents appear.

¹"Basic Properties of Real Numbers: The Real Number Line and the Real Numbers" <<http://cnx.org/content/m21895/latest/>>

²"Basic Properties of Real Numbers: Rules of Exponents" <<http://cnx.org/content/m21900/latest/>>

³"Basic Operations with Real Numbers: Absolute Value" <<http://cnx.org/content/m21876/latest/>>

⁴"Basic Operations with Real Numbers: Negative Exponents" <<http://cnx.org/content/m21882/latest/>>

⁵"Basic Operations with Real Numbers: Negative Exponents" <<http://cnx.org/content/m21882/latest/>>

Solutions to Exercises in this Module

Solution to Exercise 1 (p. 3)

$$3.46 \times 10^2$$

Solution to Exercise 2 (p. 3)

$$7.233 \times 10$$

Solution to Exercise 3 (p. 3)

$$5.3877965 \times 10^3$$

Solution to Exercise 4 (p. 3)

$$8.7 \times 10^7$$

Solution to Exercise 5 (p. 3)

$$1.79 \times 10^{20}$$

Solution to Exercise 6 (p. 3)

$$1.0 \times 10^5$$

Solution to Exercise 7 (p. 3)

$$1.0 \times 10^6$$

Solution to Exercise 8 (p. 3)

$$8.6 \times 10^{-3}$$

Solution to Exercise 9 (p. 3)

$$9.8001 \times 10^{-5}$$

Solution to Exercise 10 (p. 3)

$$5.4 \times 10^{-17}$$

Solution to Exercise 11 (p. 3)

$$1.0 \times 10^{-7}$$

Solution to Exercise 12 (p. 3)

$$1.0 \times 10^{-8}$$

Solution to Exercise 13 (p. 4)

$$925$$

Solution to Exercise 14 (p. 4)

$$401000$$

Solution to Exercise 15 (p. 4)

$$0.12$$

Solution to Exercise 16 (p. 4)

$$0.0000888$$

Solution to Exercise 17 (p. 5)

$$6 \times 10^{17}$$

Solution to Exercise 18 (p. 5)

$$6 \times 10^{20}$$

Solution to Exercise 19 (p. 5)

$$1.5 \times 10^{25}$$

Solution to Exercise 20 (p. 5)

$$6.3 \times 10^{-20}$$

Solution to Exercise 21 (p. 6)

$$5.89 \times 10^3$$

Solution to Exercise 23 (p. 6)

$$2.5 \times 10^{23}$$

Solution to Exercise 25 (p. 6)

$$2 \times 10^{-4}$$

Solution to Exercise 27 (p. 6)

$$5.7 \times 10^4$$

Solution to Exercise 29 (p. 6)

$$2.8 \times 10^{12}, 2.463 \times 10^{25}$$

Solution to Exercise 31 (p. 6)

$$3.36 \times 10^3$$

Solution to Exercise 33 (p. 6)

$$8 \times 10^6$$

Solution to Exercise 35 (p. 6)

$$1.5 \times 10^{62}$$

Solution to Exercise 37 (p. 7)

$$8 \times 10^{-9}$$

Solution to Exercise 39 (p. 7)

$$1.03 \times 10^{-18}$$

Solution to Exercise 41 (p. 7)

$$3.1 \times 10^{-26}$$

Solution to Exercise 43 (p. 7)

$$3.16 \times 10^2$$

Solution to Exercise 45 (p. 7)

$$7.4 \times 10^5$$

Solution to Exercise 47 (p. 7)

$$3.16 \times 10^{-5}$$

Solution to Exercise 49 (p. 7)

$$1 \times 10^{-16}$$

Solution to Exercise 51 (p. 7)

$$100,000,000$$

Solution to Exercise 53 (p. 7)

$$5,866,000,000,000$$

Solution to Exercise 55 (p. 7)

$$43,000,000,000,000,000,000$$

Solution to Exercise 57 (p. 8)

$$60,000,000$$

Solution to Exercise 59 (p. 8)

$$0.0000000000000000000047$$

Solution to Exercise 61 (p. 8)

$$0.0125$$

Solution to Exercise 63 (p. 8)

$$3.2 \times 10^9$$

Solution to Exercise 65 (p. 8)

$$2.4 \times 10^3$$

Solution to Exercise 67 (p. 8)

$$9 \times 10^{-16}$$

Solution to Exercise 69 (p. 8)

$$1.512 \times 10^{-31}$$

Solution to Exercise 71 (p. 8)

$$3.481 \times 10^{29}$$

Solution to Exercise 73 (p. 8)

$$7.744 \times 10^{-99}$$

Solution to Exercise 75 (p. 8)

$$6.687 \times 10^{16}$$

Solution to Exercise 77 (p. 9)

$$1.47 \times 10^{-5}$$

Solution to Exercise 79 (p. 9)

$$4.5 \times 10^{37}$$

Solution to Exercise 81 (p. 9)

$$-5, -4, -3$$

Solution to Exercise 83 (p. 9)

$$-5$$

Solution to Exercise 85 (p. 9)

$$\frac{1}{(2z+1)^2}$$